Quantum mechanics II, Chapter 2 : Entanglement (Part 2)

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Sometimes observation kills.

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Problem 1 : The quantum eraser

This question recaps the quantum eraser experiments- if you understood the lecture you can skip.

1. Calculate what is observed at the screen when the polarization sheet in Fig. 1c) is oriented at -45 degrees instead of +45 degrees. (In this case the screen only lets through $|\swarrow\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$ photons).
2. Consider the setup shown in Fig. 1d). We suppose that the photon that passes the atom flips the spin of an outer electron from $|\downarrow\rangle$ to $|\uparrow\rangle$ but is not absorbed. (For each photon that we send through the two slit experiment we use a new atom and store the previous in a quantum memory).

   (a) Calculate what is observed at the screen if the atoms are measured in basis $\{|\uparrow\rangle, |\downarrow\rangle\}$.

   (b) Calculate what is observed at the screen if the atoms are instead measured in basis $\{|\nearrow\rangle, |\swarrow\rangle\}$.
3. Discuss whether setup Fig. 1d) could be used for signalling.

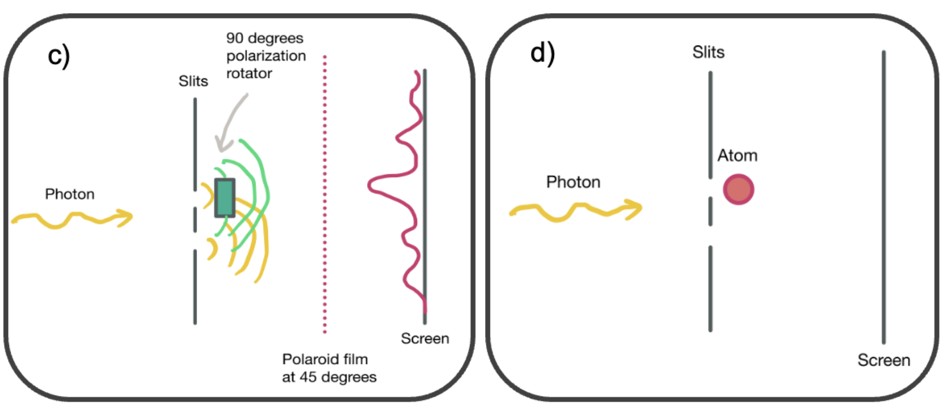

FIGURE 1 – Quantum eraser experiments

Problem 2 : No signalling

1. Suppose Alice and Bob share a two-qubit entangled state $\frac{\sqrt{3}}{2}\left|0\right\rangle_A\left|0\right\rangle_B + \frac{1}{2}\left|1\right\rangle_A\left|1\right\rangle_B$. Alice has the left qubit and Bob has the right. Alice attempts to communicate to Bob by measuring her qubit in the Z or X eigenbases.

   Compute the probability that Alice obtains each outcome, and the corresponding collapsed states of Bob's qubit.

2. Imagine Bob subsequently performs a measurement. We seek to show that the probability of him obtaining an outcome corresponding to a completely arbitrary one-qubit projector $\hat{\Pi}$ is the same regardless of whether Alice measured X or Z. That would mean he cannot locally distinguish what measurement Alice implemented, and so cannot determine her message.

   (a) Write down the probability that Bob obtains outcome λ of $\hat{\Pi}$ if Alice measures Z or X respectively, i.e. $P_B(\lambda|A \text{ measures } Z)$ and $P_B(\lambda|A \text{ measures } X)$.

   (b) Hence show that $P_B(\lambda|A \text{ measures } Z) = P_B(\lambda|A \text{ measures } X)$.

   This implies superluminal signalling by this method is impossible.

3. Consider now that Alice and Bob share an arbitrary two-qubit entangled state $|\phi_{AB}\rangle$. Show that Alice cannot superluminally communicate a bit to Bob by performing either X or Z measurements on her qubit.

Problem 3 : Bell inequality (quantum psychics version)

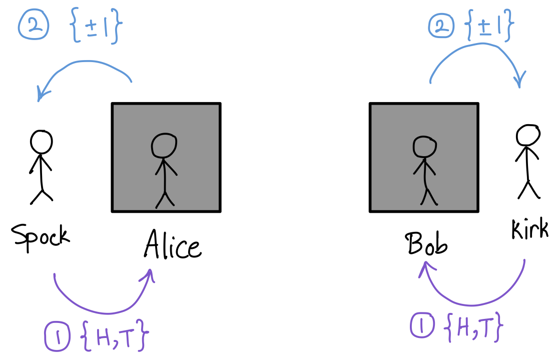

FIGURE 2 – **The Quantum Psychics Game.**

Alice and Bob claim to share a psychic connection. Some sceptics seek to test this by locking Alice and Bob into isolated rooms with no way to pass any messages between them. Outside Alice's room is a sceptic, let's call him Spock, who tosses a coin and tells Alice the outcome. Outside Bob's room is another sceptic, Kirk, who similarly tosses a coin and tells Bob the outcome. Alice and Bob must each then respond with a single bit of information ; a yes 'Y' or no 'N'. Spock and Kirk ask Alice and Bob to perform the following test :

*If Alice and Bob **both** are told 'H' they must give **opposite** answers, but **otherwise** (when one or both are told 'T') they must give the **same** answer.*

Secretly, Alice and Bob are not *psychics* - they are quantum *physicists* ! They share an entangled Bell state, $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, of which they can use the non-classical correlations stored within to pass the sceptics' test. Alice and Bob's strategy do so is as follows.

— If Alice gets told 'H' she measures in the Z basis and says 'Y' if she gets '$|0\rangle$' and 'N' if she gets '$|1\rangle$'.
— If Alice gets told 'T' she measures in the X basis and says 'Y' if she gets '$|+\rangle$' and 'N' if she gets '$|-\rangle$'.
— If Bob gets told 'H' he measures in the basis

$$\left\{|h\rangle = \sin(\pi/8)|0\rangle + \cos(\pi/8)|1\rangle \, , \, |\bar{h}\rangle = \cos(\pi/8)|0\rangle - \sin(\pi/8)|1\rangle\right\} \tag{1}$$

and says 'Y' if he gets '$|h\rangle$' and 'N' if she gets '$|\bar{h}\rangle$'.
— If Bob gets told 'T' he measures in the basis

$$\left\{|t\rangle = \cos(\pi/8)|0\rangle + \sin(\pi/8)|1\rangle \, , \, |\bar{t}\rangle = \sin(\pi/8)|0\rangle - \cos(\pi/8)|1\rangle\right\} \tag{2}$$

and says 'Y' if he gets '$|t\rangle$' and 'N' if she gets '$|\bar{t}\rangle$'.

Show that Alice and Bob can win the test and fool the sceptics with probability

$$P_{\text{Quantum}} = \cos(\pi/8)^2 = \frac{2+\sqrt{2}}{4} \approx 0.854 \, . \tag{3}$$